Biot-Savart law: $\vec{B}(\vec{r}_f) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{\ell} \times \vec{r}_{sf}}{r_{sf}^2}, \vec{r}_{sf} = \vec{r}_f - \vec{r}_s$.

Field on axis of current loop. Dipolar with $B \propto \frac{I}{z^3}, z >> R$.

Ferromagnet due to aligned electron spins, each with $\mu \approx \frac{e\hbar}{2m_e}, \hbar = \frac{h}{2\pi}$; $\vec{B} \approx \mu_0 \vec{M}$, with $\vec{M}$ being the magnetic dipole moment per volume.

IX Electromagnetic Induction and Faraday's Law

A Faraday's Law

1 Magnetic Flux

Magnetic flux – similar to electric flux. $\Phi_E = \int \vec{E} \cdot d\vec{A}$; but for magnetic flux, $\Phi_B = \int \vec{B} \cdot d\vec{A}$.

Gauss said that $\oiint \vec{E} \cdot d\vec{A} = \frac{Q_{encl}}{\epsilon_0}$; but if we were to try that with a magnetic field, $\oiint \vec{B} \cdot d\vec{A} = 0$, because there are no magnetic monopoles.

When we define the magnetic flux, that can, of course, be any area; a closed area, however, will always return 0.

2 Faraday's Law

A time-varying magnetic field *induces* an electric field. Symbolically:

$$\xi = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

$(\vec{v} = \vec{0})$

Suppose two neighboring circuits, (1), and (2); a magnet (M) is positioned near (1):

1 If we move (M) towards (1), this will change the flux in (1), which implies (in turn) a change in ξ.

2 Changing the current in (1) induces an EMF in (2).

3 Moving (1) will induce $\vec{E}$ in (2).

Note that the induced electric field is *not* conservative, which is completely contrary to electrostatic fields – this is because we have

$$\oint \vec{E} \cdot d\vec{\ell} = -\frac{d\Phi_B}{dt}$$

...which depends on time (obviously) – $\frac{d\Phi_B}{dt} = 0$ for a static case.

Ex. Problem 29-16

We have a high-voltage line running overhead with $I = 55$ A; we put a loop of length ℓ underneath it and want to set ℓ such that $V_{rms} = 120$ V $\rightarrow V_0 = 170$ V. To make it easier, we wrap the loop 10 times ($N = 10$).

First, calculate the flux: $B = \frac{\mu_0 I}{2\pi r}$ for a wire, so

$$\Phi_B = \iint \vec{B} \cdot d\vec{A} = \int \frac{\mu_0 I}{2\pi} \ell \frac{dr}{r} = \frac{\mu_0 I \ell}{2\pi} \ln\left(\frac{r_2}{r_1}\right)$$

Since the current is oscillating back and forth, we have a magnetic flux

$$\Phi_B = \Phi_{B0} \cos(\omega t) = \frac{\mu_0 I_0 \ell \ln(1.4)}{2\pi} \cos(\omega t)$$

$$\xi = -N\frac{d\Phi_B}{dt} = \frac{N\omega\mu_0 I_0 \ell \ln(1.4)}{2\pi} \sin(\omega t)$$

Recall that $\omega = 2\pi f, f = 60$ Hz $\Rightarrow \omega = 120\pi$;

So we plug in $\xi = 170$ V, we get that $\ell = 12$ m. For this reason, power companies keep careful track of things like this.

B Motional EMFs

Two experiments: experiment one sees a wire loop with a magnet being moved further away – the movement changes Φ_B, which creates a ξ. In experiment two, we leave the magnet stationary – the movement does *not* change the electric field, so $\vec{E}$ stays 0. However, there will still be the same ξ, of course – the change in the flux is the same, as far as the loop is concerned.

It may be helpful to remember that the EMF $\xi = \oint \frac{\vec{F}}{Q} \cdot d\vec{\ell}$ – for $\vec{E} + \vec{B}$, we need *total* force – $\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B}$, so

$$\xi = \oint (\vec{E} + \vec{v} \times \vec{B}) \cdot d\vec{\ell}$$

Remember that when we spoke of Faraday's law, $\vec{v} = \vec{0}$, but in experiment 2, we have what is known as the **motional EMF** – $\oint \vec{v} \times \vec{B} \cdot d\vec{\ell}$.

As an example, we have a half-loop of magnetic wires with a rod laying across the wires, with $\vec{B}$ going into the board. We apply force to make the rod move to the right; therefore the induced current will flow upwards (and counterclockwise).

C Lenz's Law

Note that there is a negative sign before $\frac{d\Phi_B}{dt}$ – Lenz said that **the induced EMF opposes the change that caused it** ('it' being the induced EMF).

Suppose a loop with a magnetic field threading it; if B is increased, the current that will occur in the loop is *opposite* to the current that would induce a greater magnetic field – so an upwards field increasing will produce a clockwise current.

Otherwise, we would have a magnetic field creating a current, which would in turn increase the magnetic field more! This is known as *the Helvetica Scenario*.